Optimal tolerance re-allocation for the generative process sequence

UTPAL ROY and YING-CHE FANG

Knowledge Based Engineering Laboratory, Department of Mechanical, Aerospace and Manufacturing Engineering, Syracuse University, Syracuse, New York 13244, USA

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1. Introduction

Because manufacturing processes introduce imperfections, tolerances are assigned to parts so that when they are assembled the design requirements can be realized. However, how the tolerances are distributed to each dimension for a specified stack-up condition has a significant impact on the total cost. The optimal tolerance selection (for critical part dimensions) by associating the costs, which are the functions of tolerance values and processes involved, have been discussed in many research works (Balas, 1965; Cagan and Kurfess, 1992; Chase and Greenwood, 1988; Chase et al., 1990; Dong and Hu, 1991; Hasofer and Lind, 1974; Lee and Woo, 1989; Ostwald and Huang, 1977; Speckhart, 1972; Spotts, 1973; Wu et al., 1988; Zhang and Wang, 1993).

Ostwald and Huang (1977) formulated the optimization of tolerance as linear programs with variables assuming the value of zero or one. The particular algorithm is based on that presented by Balas (1965). Some specific tolerance values are determined by some available machining process sequences. The cost associated with the tolerance values and process sequences are also determined. The minimal cost is the objective function, and the design requirements are used as constraints. By solving the zero–one algorithm, the minimal cost can be achieved by selecting the available tolerance values.

The tolerance values are discrete and the process sequences are also determined, it is suitable in the situation where the process sequence and the tolerance of the sequence is fixed.

Another discrete model is presented by Lee and Woo (1989). The cost-tolerance model that they use in their work is similar to Huang and Ostwald’s. They also treat a tolerance that is unique for a given process. However, they simplify the complex stack-up condition using the reliability index, which was previously advocated by Hasofer and Lind (1974), and develop a branch-and-bound algorithm for efficient tree enumeration. Monotonicity between the tolerance and cost, and monotonicity between tolerance and reliability index, are observed to make the enumeration tree small. Because of the simplification of the stack-up condition and the efficient tree enumeration method, the applicability of this model is significantly increased.

Chase and Greenwood (1988) and Chase et al. (1990) use a continuous cost-tolerance model to solve the tolerance optimization problem. They enhance the method of the Lagrange multiplier, which was first applied in Spotts’ (1973) model, to multiprocess selection. They use a continuous cost function based on a fixed process sequence. It lacks the flexibility of considering different process plans. Therefore the global optimal solution may not always be achieved.

In the above works, either only one process or a predetermined multiple-process sequence is involved in consideration of optimal tolerance selection. The concept of generative process planning is not incorporated in
tolerance cost analysis. Dong and Hu (1991) propose a way to decide the optimal process sequence on the basis of a given tolerance. However, the tolerance selection cannot be achieved. It should be noted that the allocation or re-allocation of tolerances is inherently highly iterative in nature. The selection of manufacturing processes depends on the tolerance values, and the optimal tolerance values are in turn decided by the processes selected. Therefore it is impossible to attain an optimal target value for a tolerance in only one step. Process capabilities of available manufacturing facilities add another important dimension to this tolerance allocation/re-allocation problem, because a process plan can never be simply decided without considering the process capabilities. In this paper, a new methodology is proposed to incorporate both the process capabilities and generative process sequence plan into the re-allocation of tolerance. It provides a framework for determining the tolerance values and its corresponding process sequence in an iterative fashion such that the minimal overall manufacturing cost can be obtained. Continuous tolerance-cost functions, in spite of an increase in their complexity compared with discrete models, are adopted to achieve better accuracy in cost estimation.

2. Preliminary

In the manufacturing industry it normally takes more than one process to produce a single part. For example, a typical process sequence for producing a rotational surface includes: (a) forging, (b) rough turning, (c) semi-finish turning, and (d) finish grinding. However, there may be more than one possible sequence available. An alternative process sequence for the same part may be: (a) die casting, (b) semi-finishing turning, and (c) finish turning. Therefore by selecting the best sequence and the point to change from one process to another (e.g., the point to change the process from turning to grinding), the optimal cost for a given tolerance can then be achieved.

Given a list of available machining processes, there is a finite number of sequences that can be decided. Fig. 1 shows some processes with their tolerance ranges. Some of the possible sequences to produce a part with a tolerance ranging from $40 \times 10^{-3}$ to $0.7 \times 10^{-3}$ are:

- $FC \rightarrow DG \rightarrow Dr \rightarrow RB \rightarrow Ho$;
- $DG \rightarrow Dr \rightarrow RB$;
- $DG \rightarrow RB$.

Fig. 2 shows a simple stack-up condition for a typical block assembly with the functional requirement:

$$U \geq \text{clearance} \geq L$$

The problem to be solved is to assign tolerance values to each component and determine its corresponding process sequence to minimize the total cost. The initial tolerances and process sequences for each component can be arbitrarily determined or given by designers so they have better starting values. For each possible process sequence, the point of interest is to attain a tolerance value and the final process in the sequence to produce this tolerance such that it has the minimum cost. This process selection can be formulated as a zero-one algorithm. Owing to the continuous cost function, nonlinear terms can be expected in the objective function. The constraints, in addition to functional requirements and the single final process selection in the zero-one algorithm (Balas, 1965), includes the capabilities of each process. The tolerance re-allocation problem can therefore be formulated as a mixed-integer nonlinear programming problem. The Lagrange multiplier (Spotts, 1973) method and exhaustive search are utilized to solve this problem. Before the re-allocation problem formulation is discussed further, there follows a list of the notations used throughout the rest of this paper.

- $A_{ij}$: setup cost for process $i$ dimension $j$;
- $B_{ij}$: cost of producing a single component to a specified tolerance for dimension $j$ by process $i$;
- $C_{il}$: minimum cost by applying process $i$;
- $C_{Uj}$: maximum cost by applying process $i$;
- $D_j$: the magnitude of dimension $j$;
- $EP_{ij}$: the Equivalent Point between process $i-1$ and $i$. 

Figure 1 Tolerance ranges for certain processes.

Figure 2 A simple stack-up condition.
3. Tolerance re-allocation methodology

3.1. Process cost versus tolerance

Fig. 3 is a typical cost-tolerance relations chart for a specific process sequence. The horizontal axis is the tolerance value and the vertical axis represents the cost. \( T_j \) is the original tolerance assigned by the designer for dimension \( j \), and \( T'_j \) is the final tolerance value after the re-allocation process for the current process sequence. \( EP \) points in the figure stand for Equivalent Point. Its meaning and significance will be discussed in detail in Section 3.2. Many cost functions associated with tolerance have been proposed over the years (Chase and Greenwood, 1988; Michael and Siddall, 1982; Sutherland and Roth, 1975; Speckhart, 1972; Spotts, 1973). Table 1 summarizes these functions. In this study, the Reciprocal Model (Chase and Greenwood, 1988) is adopted to formulate the reallocation functions because of its simple form, while its accuracy is also acceptable (Wu et al., 1988). The other cost functions, however, can also be used in reallocation functions in the following formulation, as discussed in the next few sections.

It should be noted from the cost-tolerance curves characteristic (Fig. 3) that for a machining process with a higher capability of generating a precision surface, its constant cost level (when tolerance \( \rightarrow \infty \)) will generally be higher (Bjorke, 1989). In other words, for the Reciprocal Model,

\[
A_{ij} + \frac{B_{ij}}{T_j},
\]

this constant cost level will be the setup cost \( A_{ij} \). Nevertheless, the production cost will decrease according to the increase in high-precision capability. Therefore the production cost \( B_{ij} \) will be smaller for a higher capability of generating a precision surface.

Table 1. Cost-tolerance models

<table>
<thead>
<tr>
<th>Model name</th>
<th>Model</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sutherland</td>
<td>( BT^{-A} )</td>
<td>Sutherland and Roth (1975)</td>
</tr>
<tr>
<td>Reciprocal square</td>
<td>( A + B/T^2 )</td>
<td>Spotts (1973)</td>
</tr>
<tr>
<td>Reciprocal</td>
<td>( A + B/T )</td>
<td>Chase and Greenwood (1988)</td>
</tr>
<tr>
<td>Exponential</td>
<td>( Ae^{-BT} )</td>
<td>Speckhart (1972)</td>
</tr>
<tr>
<td>Michael–Siddall</td>
<td>( At^{-B_e^{-dT}} )</td>
<td>Michael and Siddall (1982)</td>
</tr>
</tbody>
</table>

3.2. Tolerance re-allocation cost function

The original tolerances, which are assigned by designers on the basis of handbooks and experience, cannot always be expected to be optimal or even feasible, because they may yield an unacceptable manufacturing cost. To achieve a lower manufacturing cost, the tolerances associated with higher manufacturing costs should be relaxed while the tolerances associated with lower manufacturing costs are tightened. Because cost functions are decided by the process sequences, the process sequence is selected on the basis of the originally assigned tolerances at the start of the re-allocation process. The results of the first re-allocation (i.e., the modified tolerance) are then used as inputs to determine the second process sequence, because the process sequences based on the original tolerances may not be optimal for the modified tolerances. The cost functions of the second process sequence are used for the second iteration of the re-allocation process. The optimal process sequence and tolerance values can be obtained by repeating this iterative procedure. The iteration process terminates with an optimal solution when the results of two consecutive iterations become the same.

In Fig. 3, \( T_j \) is the originally assigned tolerance for dimension \( j \). In the re-allocation process, the tolerance \( T_j \) can either move up or move down along the x-axis. The cost difference resulting from this movement then needs to be calculated to verify whether or not the overall objective function is improved.

In the re-allocation process, the re-allocated tolerance can either remain within the range of the same machin-
ing process as the re-allocation started, or it may shift to other processes. For the first case, the cost difference (for dimension \( j \)), \( \Delta TC_{ij} \), due to the re-allocation can be calculated as

\[
\Delta TC_{(i,j|y)} = B_{ij}\left(\frac{1}{T_{j}^*} - \frac{1}{T_{j}}\right).
\]

(4)

It is important to note that (4) represents the cost difference corresponding to the tolerance \( T_{j} \) and \( T_{j}^* \), but not the total cost. The part is initially produced from some arbitrary tolerance \( T_0 \) to \( T_{j} \). The total cost of dimension \( j \) for process \( i \), \( TC_{ij} \), for producing a part from \( T_0 \) to \( T_{j} \) should be calculated as

\[
TC_{ij} = A_{ij} + B_{ij}\left(\frac{1}{T_{j}} - \frac{1}{T_{0}}\right).
\]

(5)

If the tolerance shifts from its present process to any upper-grade process or lower-grade process (e.g., in Fig. 3, if present tolerance \( T_{j} \) moves to \( T_{j}^* \)), process 3 also becomes eligible along with process 2; here the tolerance shifts to an upper-grade process), the situation becomes more complicated. The setup cost \( A_{ij} \) then has to be incorporated as a component in the cost difference calculation. Considering process 2 and process 3 in Fig. 3, in improving the tolerance from \( T_{j} \) to \( T_{j}^* \), the first best point to switch process from 2 to 3 may be \( T_{0} \), which is the largest tolerance produced by process 3. Because \( B_{0j} \) is less than \( B_{2j} \) (as in (3)), the same tolerance difference, process 3 will cost less than process 2. However, moving to a new manufacturing process requires the consideration of setup cost in the cost difference calculation. If the saving of a new process cannot offset its setup cost, it will be a good idea to stay in the same process. However, if the saving is large enough, it would be wise to switch to the new process right at the point where the process becomes available (e.g., the point \( C_{iL} \) in Fig. 3). The decision of whether to change or not to change a process can be determined at the Equivalent Point (\( EP \)).

The \( EP_{ij} \) is a tolerance value that has an equivalent cost for either switching process to \( i \) or staying in process \( i - 1 \) for dimension \( j \). Therefore if the re-allocated tolerance is less than \( EP_{ij} \), process \( i \) should be adopted immediately when it is available; otherwise it should stay in the same process.

The value of \( EP \) can be derived as follows. In Fig. 3, the cost difference between points \( EP_{ij} \) and \( C_{iL} \) is

\[
\Delta TC_{(2,3;ij)} = B_{2j}\left(\frac{1}{EP_{ij}} - \frac{1}{C_{iL}}\right)
\]

if the decision is to stay in process 2, or

\[
\Delta TC_{(2,3;ij)} = B_{3j}\left(\frac{1}{EP_{ij}} - \frac{1}{C_{iL}}\right) + A_{3j}
\]

if the decision is to shift to process 3. Therefore \( EP_{ij} \) can be derived as

\[
EP_{ij} = \frac{(B_{2j} - B_{3j})C_{iL}}{A_{ij}C_{iL} + B_{2j} - B_{3j}}
\]

or expressed in the general form

\[
EP_{ij} = \frac{(B_{i - 1,j} - B_{ij})C_{iL}}{A_{ij}C_{iL} + B_{i - 1,j} - B_{ij}}.
\]

(6)

The cost difference for re-allocating the tolerance of dimension \( j \) from \( T_{j} \) to \( T_{j}^* \) between process 2 and process 3 becomes

\[
\Delta TC_{(2,3;ij)} = B_{3j}\left(\frac{1}{T_{j}^*} - \frac{1}{C_{iL}}\right) + B_{2j}\left(\frac{1}{C_{iL}} - \frac{1}{T_{j}}\right) + A_{3j}
\]

The general form for the cost difference when the tolerance \( T_{j} \) moves from process \( i \) to process \( i + k \) is

\[
\Delta TC_{(i,j + k)} = B_{i + k}\left(\frac{1}{T_{j}^*} - \frac{1}{C_{(i + k)L}}\right) + \sum_{d=1}^{k-1} B_{i + d,j}\left(\frac{1}{C_{(i + d)L}} - \frac{1}{C_{(i + d - 1)L}}\right) + B_{ij}\left(\frac{1}{C_{iL}} - \frac{1}{T_{j}}\right) + \sum_{n=i}^{i+k} A_{nj}.
\]

(7)

If the re-allocated tolerance shifts to a lower-grade process (from process 2 to process 1 as in Fig. 3), the cost difference function can be derived in a similar fashion, and it can be expressed as

\[
\Delta TC_{(i,j - k)} = B_{i - k}\left(\frac{1}{T_{j}} - \frac{1}{C_{(i - k) + 1L}}\right) + \sum_{d=1}^{k-1} B_{i - d,j}\left(\frac{1}{C_{(i - d) + 1L}} - \frac{1}{C_{(i - d) - 1L}}\right) + B_{ij}\left(\frac{1}{C_{iL}} - \frac{1}{T_{j}}\right) - \sum_{n=i}^{i+k} A_{nj}.
\]

(8)

There is still one more constraint to consider: the constraint of process capabilities of the available manufacturing facilities. Capability, in this study, means the range of tolerance values that a process can generate. Fig. 4 shows the capability ranges of four processes and their corresponding \( EP \) points.

If process \( i \) is the final process where the re-allocated tolerance \( T_{j}^* \) should be produced, it should meet the following two constraints:

\[
C_{iL} \leq T_{j} \leq C_{iL};
\]

\[
EP_{i + 1} \leq T_{j} \leq EP_{ij}.
\]
These two constraints can be combined and represented as follows:

\[
\max (EP_{i+1,j}, C_{iU}) \leq T_j \leq \min (EP_{i,j}, C_{iL}). \tag{9}
\]

The procedure to select the final process and its cost difference is summarized in Table 2. The initial final process \(i\) in Table 2 is the final process of the original process sequence. After the re-allocation, the tolerance value changes to \(T_f\) and the final process becomes \(i+k\) if \(\max (EP_{i+k+1,j}, C_{i+k}L) \leq T_f \leq \min (EP_{i+k,j}, C_{i+k}L)\). The cost difference, \(\Delta TC_{(i+k)j}\), can therefore be determined.

### 3.3. Objective function for tolerance re-allocation

For the simple stack-up condition as shown in Fig. 2, there are \(n-1\) blocks to be assembled into a slot. The objective function is to minimize the total cost by selecting proper final processes for each dimension and then minimize the cost difference functions ((7) and (8)) associated with the processes. Let \(P_{ij}\) be the processes in the process sequence to produce dimension \(j\), where \(i = 1\ldots m\), and \(P_{kj}\) be the final process. The objective function can then be formulated as

\[
\sum_{j=1}^{n} \sum_{i=1}^{m} Q_{ij} \Delta TC_{(P_{ij}, P_{kj})j}, \tag{10}
\]

where \(Q_{ij} \in \{0,1\}\) equals one if the process \(i\) is the final

### Table 2. The criterion for selecting cost type and final process

<table>
<thead>
<tr>
<th>Initial tolerance</th>
<th>(T_j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial final process</td>
<td>Process (i)</td>
</tr>
<tr>
<td>Re-allocated tolerance</td>
<td>(T_f^j)</td>
</tr>
<tr>
<td>Situation</td>
<td>(\max (EP_{i+k+1,j}, C_{i+k}L) \leq T_f^j \leq \min (EP_{i+k,j}, C_{i+k}L))</td>
</tr>
<tr>
<td>Cost difference</td>
<td>(\Delta TC_{(i+k)j})</td>
</tr>
<tr>
<td>Final process</td>
<td>Process (i+k)</td>
</tr>
</tbody>
</table>

process for dimension \(j\), otherwise it equals zero. Because there is exactly one final process for each dimension, the following constraints are true:

\[
\sum_{i=1}^{m} Q_{ij} = 1, \, j = 1, \ldots, n. \tag{11}
\]

The final tolerance assignments have to meet the functional requirement, which is

\[
U \geq \text{clearance} \geq L.
\]

The smallest possible clearance occurs when

\[
\text{clearance} = D_n + T_f^j - \sum_{j=1}^{n-1} (D_j + T_j) \geq L, \tag{12}
\]

and the largest possible clearance is

\[
\text{clearance} = D_n + T_f^j - \sum_{j=1}^{n-1} (D_j + T_j) \leq U. \tag{13}
\]

Let the tolerance shift of the dimension \(j\), \(t_j = T_f^j - T_j\), then (12) and (13) can be rewritten as

\[
\sum_{j=1}^{n} t_j \leq T_L \tag{14}
\]

and

\[
\sum_{j=1}^{n} t_j \leq T_U \tag{15}
\]

respectively, where

Figure 4 The capability ranges and EP points.

Figure 5 The simple stack up condition.
Table 3. Summary of the objective function and constraints for the tolerance re-allocation problem

| Objective function | \( \sum_{j=1}^{n} \sum_{i=1}^{m} Q_{ij} \Delta TC\left(p_{ki}, p_{pi}\right) \) |
| Constraints | \( \sum_{j=1}^{n} Q_{ij} = 1, j = 1, \ldots, n \) |
| Constraints | \( \sum_{j=1}^{n} t_j \leq T_{\text{min}} \) |
| Constraints | \( \max(EP\left(p_{ki} + 1\right), C_{pi}) \leq T_{j}^{f} \leq \min(EP\left(p_{ki}\right), C_{pi}) \) |

\[
T_L = D_n - \sum_{j=1}^{n-1} D_j - \sum_{j=1}^{n} T_j - L
\]

and

\[
T_U = U - D_n - \sum_{j=1}^{n-1} D_j - \sum_{j=1}^{n} T_j.
\]

Therefore for the final tolerance value that will be able to meet the functional requirement, the sum of tolerance shifts have to satisfy

\[
\sum_{j=1}^{n} t_j \leq T_{\text{min}}, \quad (16)
\]

where \( T_{\text{min}} = \min(T_U, T_L) \).

For the re-allocated tolerances and their associated final processes to be valid, (9) has to be satisfied. Let \( P_{ij} \) be the final process for the re-allocated tolerance \( T_j^{f} \); (9) can be rewritten as

\[
\max(EP\left(p_{ki} + 1\right), C_{pi}) \leq T_{j}^{f} = (T_j + t_j) \leq \min(EP\left(p_{ki}\right), C_{pi}).
\]

Table 3 summarizes the objective function and all the constraints for the tolerance re-allocation problem.

4. Solution procedures

The closed-form solution for the least-cost component by the Lagrange multiplier method was developed by Spotts (1973). By introducing a Lagrange multiplier, the objective function (10) and the constraint (16) can be formulated as follows:

\[
F(t_j) = \sum_{j=1}^{n} \sum_{i=1}^{m} Q_{ij} \Delta TC\left(p_{ki}, p_{pi}\right) + \lambda \left( \sum_{j=1}^{n} t_j - T_{\text{min}} \right) = 0.
\]

Taking partial differentiation with respect to \( t_j, j = 1, \ldots, n, \)

\[
\frac{\partial F(t_j)}{\partial t_j} = 0
\]

results in:

\[
\lambda = \sum_{i=1}^{m} Q_{ij} B_{ij} / \left( T_j^{f} \right)^2.
\]

\( \lambda \) can be eliminated by expressing it in terms of \( T_j^{f} \); then

\[
\sum_{i=1}^{m} Q_{ij} B_{ij} / \left( T_j^{f} \right)^2 = \sum_{i=1}^{n} Q_{ij} B_{ij} / \left( T_j^{f} \right)^2.
\]

Replacing \( T_j^{f} \) by \( T_j + t_j \),

\[
\frac{T_j + t_j}{T_j + t_1} = \sqrt{\frac{\sum_{i=1}^{m} Q_{ij} B_{ij}}{\sum_{i=1}^{n} Q_{ij} B_{ij} - \sum_{i=1}^{m} Q_{ij} B_{ij}}}
\]

Therefore the deviation of tolerance in dimension \( j, t_j \), can be expressed as

\[
t_j = \sqrt{\frac{\sum_{i=1}^{m} Q_{ij} B_{ij}}{\sum_{i=1}^{n} Q_{ij} B_{ij} - \sum_{i=1}^{m} Q_{ij} B_{ij}}}(T_j + t_1) - T_j
\]

and \( t_1 \) can be determined by applying (16):

\[
\sum_{j=1}^{n} t_j = t_1 + (T_j + t_1) \left( \frac{\sum_{j=1}^{m} Q_{ij} B_{ij}}{\sum_{i=1}^{n} Q_{ij} B_{ij}} \right) - T_j = \sum_{j=2}^{n} T_{\text{min}};
\]

therefore

\[
T_{\text{min}} + \sum_{j=2}^{n} T_j - T_j \sum_{j=2}^{n} \sqrt{\frac{\sum_{i=1}^{m} Q_{ij} B_{ij}}{\sum_{i=1}^{n} Q_{ij} B_{ij} - \sum_{i=1}^{m} Q_{ij} B_{ij}}}
\]

4.1. Exhaustive search

In (10) the value of \( Q_{ij} \in \{0, 1\} \) is not yet determined. Furthermore, for the resulting process and the tolerance to be valid, (9) has to be satisfied. Because the cost functions are nonlinear and costs keep changing with the modified tolerance, there is no apparent trend between the modified tolerances and total cost difference. Methods such as branch-and-bound cannot be applied here. The univariate method described by Chase and Greenwood (1988) and Chase et al. (1990) may be used. However, the global optimum is not guaranteed. The exhaustive search, in spite of its verbosity, is still the safest way to obtain the global optimal value. The complexity of the algorithm is \( O(n^2) \), where \( n \) is the total number of dimensions (or processes).

The overall procedure for solving this mixed-integer nonlinear programming problem is illustrated below. In step 1, all the possible combinations of processes are used to compute the re-allocated tolerances. Step 2 checks the feasibility of these tolerances for each dimension. For each set of tolerances that are feasible, the total cost can be derived as in step 3. The minimum cost and its corresponding processes are recorded and returned as the final result.
Step 1. Select one process for each dimension from 1 to \( j \) and calculate \( t_j^l \) and \( T_f^j \) by applying (17) and (18).

Step 2. Verify whether \( \max(EP(P_0 + 1/2),C_{P_0}) \leq T_f^j \leq \min(EP(P_0 + 1/2),C_{P_0}) \), where \( P_0 \) is the selected final process for dimension \( j \) in step 1.

Step 3. Obtain the total cost by applying (10).

Step 4. Record the minimal total cost from step 3 along with its corresponding processes.

5. Case study

We consider the same block example as shown in Fig. 3 and specify particular dimensions and tolerances (Fig. 5). The original process sequences for each block are listed in Table 4. Initially the specified tolerance value for Block 1 is \( 1.9 \times 10^{-3} \); for Block 2, \( 2.1 \times 10^{-3} \); for Block 3, \( 2.4 \times 10^{-3} \); and for slot, \( 3.3 \times 10^{-3} \). Therefore, from Table 3, it is evident that the block 1 needs only the process sequence \{Disk-filing, Shaping\}, Block 2 needs \{Disk-filing, Shaping\}, Block 3 needs \{Disk-grinding, Milling\} and slot needs \{Disk-grinding, Milling\}. The objective function is formulated as in (10) and the constraints are shown as in (9), (11), and (16). With the specified tolerance values and \( T_U = T_L = -1.7 \times 10^{-3} \), it is evident that the three blocks are assembled loosely in the slot. Therefore we try to modify the tolerance values (e.g., \( TO_1, TO_2, TO_3, TO_4 \)) in an economic way.

The parameters of cost function for each process are stored in the database and can be retrieved whenever needed. The capabilities of each process are also stored in the database. The functional requirement is

\[-8 \times 10^{-3} \leq \text{clearance} \leq 8 \times 10^{-3}\]

The re-allocation result is listed in Table 5. The shift of tolerance for the slot can be explained with help of Fig. 6. The original process sequence for the slot is \{Disk-grinding, Milling\} and the tolerance given is \( 3.3 \times 10^{-3} \). After reallocation the modified tolerance becomes \( 1.16 \times 10^{-3} \). Because the tolerance value is lower than the \( EP \) point 1.8, the final process will be grinding. It means that the slot needs to have the final sequence as \{Disk-grinding, Milling, Grinding\}.

Although final tolerance values for Blocks 1, 2, and 3 have changed, their final processes have not. This means that all the blocks retain their initial process sequence (e.g., for Block 1 it is \{Disk-filing, Shaping\}).

6. Conclusion

In this paper a new methodology has been proposed for the tolerance re-allocation problem. It provides a framework for determining optimal tolerance values and its corresponding process sequences for the most economical manufacturing. The methodology considers both the process capabilities (of the manufacturing processes involved) and the generative process sequence plots into the re-allocation of tolerances and it formulates a mixed integer nonlinear programming model for the tolerance re-allocation problem. The proposed technique has been applied to the tolerance re-allocation problem of a system of simple blocks in a slot (Fig. 5), considering the manufacturing processes of shaping, milling, and grinding. This simple example has been used to demon-

![Figure 6 The cost function and tolerance shift.](image)

**Table 4. The process sequence, cost function parameters and capabilities**

<table>
<thead>
<tr>
<th>Part name sequence</th>
<th>Cost function parameters, ( A_{ij}, B_{ij} \times 10^{-3} )</th>
<th>Capabilities, ( 10^{-3} \times C_{ijL}, 10^{-3} \times C_{ijU} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block 1 Disk-filing Shaping 20, 85 10, 20</td>
<td>1.8, 10</td>
<td>0.05, 0.2</td>
</tr>
<tr>
<td></td>
<td>1.8, 10</td>
<td>0.05, 0.2</td>
</tr>
<tr>
<td>Block 2 Disk-filing Shaping 20, 85 10, 20</td>
<td>10, 20</td>
<td>0.18, 1.8</td>
</tr>
<tr>
<td></td>
<td>1.8, 10</td>
<td>0.18, 1.8</td>
</tr>
<tr>
<td>Block 3 Disk-grinding Milling 15, 90 10, 20</td>
<td>10, 20</td>
<td>1.523</td>
</tr>
<tr>
<td></td>
<td>1.8, 10</td>
<td>1.523</td>
</tr>
<tr>
<td>Slot Disk-grinding Milling 15, 90 10, 20</td>
<td>10, 20</td>
<td>134.175</td>
</tr>
<tr>
<td></td>
<td>1.8, 10</td>
<td>134.175</td>
</tr>
<tr>
<td></td>
<td></td>
<td>95.546</td>
</tr>
</tbody>
</table>

**Table 5. The result of reallocation**

<table>
<thead>
<tr>
<th>Part name</th>
<th>Dimension (inch)</th>
<th>Original tol. value</th>
<th>Modified tol. value</th>
<th>Final process</th>
<th>Cost difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block 1</td>
<td>1.50</td>
<td>1.9</td>
<td>2.40</td>
<td>Shaping</td>
<td>−37.68</td>
</tr>
<tr>
<td>Block 2</td>
<td>2.25</td>
<td>2.1</td>
<td>2.25</td>
<td>Shaping</td>
<td>−2.984</td>
</tr>
<tr>
<td>Block 3</td>
<td>1.25</td>
<td>2.4</td>
<td>2.18</td>
<td>Milling</td>
<td>1.523</td>
</tr>
<tr>
<td>Slot</td>
<td>5.00</td>
<td>3.3</td>
<td>1.16</td>
<td>Grinding</td>
<td>134.175</td>
</tr>
<tr>
<td>Total cost difference</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>95.546</td>
</tr>
</tbody>
</table>
strate the theory. However, it has brought out many issues involved in formulating appropriate models. The present model does not take into account the availability of machines, machine loading and other scheduling constraints. These important constraints will be considered in future to update the proposed model.

The reported work is a part of a computer-aided tolerance analysis and synthesis system. The necessary information on parts geometry and functions for the analysis is directly retrieved from the CAD data models of parts. This tolerance analysis and synthesis module will be further integrated with a generative CAPP (Computer-Aided Process Planning) system to develop a truly integrated design environment.

Acknowledgement

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References


Biographies

Utpal Roy is an Associate Professor at the Mechanical, Aerospace and Manufacturing Engineering Department and the Director of the Multi-Disciplinary Analysis and Design Laboratory (MADLAB) at Syracuse University, Syracuse, New York. He holds the Bachelor (1978) and M.S. degrees in Mechanical Engineering and the Ph.D. (1989) degree in Industrial Engineering. Dr Roy’s research interests are in Computer Aided Design, Computer Integrated Manufacturing and Artificial Intelligence applications. He teaches Computer Aided Design and Manufacturing related courses.

Ying-che Fang is currently a Ph.D. candidate in the Department of Industrial and Operations Engineering at the University of Michigan. He received his B.S. degree in civil engineering and management science from the Chiao Tung University at Hsinchu, Taiwan, in 1989 and his M.S. degree in manufacturing engineering from Syracuse University in 1993. His primary research interests are in the areas of CAD/CAM, computational geometry, and geometric modeling.